Low-momentum Hyperon-Nucleon Interactions

B.-J. Schaefer,[1,*] M. Wagner,[1] J. Wambach,[1,2] T.T.S. Kuo,[3] and G.E. Brown[3]

[1] Institut für Kernphysik, TU Darmstadt, D-64289 Darmstadt, Germany
[2] Gesellschaft für Schwerionenforschung GSI, D-64291 Darmstadt, Germany
[3] Department of Physics and Astronomy, State University of New York, Stony Brook, NY 11794-3800, USA

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We present a first exploratory study for hyperon-nucleon interactions using renormalization group techniques. The effective two-body low-momentum potential $V_{\text{low } k}$ is obtained by integrating out the high-momentum components from realistic Nijmegen YN potentials. A T-matrix equivalence approach is employed, so that the low-energy phase shifts are reproduced by $V_{\text{low } k}$ up to a momentum scale $\Lambda \sim 500$ MeV. Although the various bare Nijmegen models differ somewhat from each other, the corresponding $V_{\text{low } k}$ interactions show convergence in some channels, suggesting a possible unique YN interaction at low momenta.

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Starting from modern nucleon-nucleon interactions and performing a renormalization group (RG) decimation it has become possible to derive a unique low-momentum effective interaction $V_{\text{low } k}$ [1]. The basic idea is to integrate out the short-distance physics encoded in hard-core interactions which are not well constrained by the available phase shift data. The resulting effective interactions form the starting point for ab-initio nuclear structure calculations in few-body systems [2], shell-model studies [3] and mean-field treatments via density-functional methods [4]. They also serve as input for the derivation of Landau Fermi-liquid interactions and provide predictions of pairing gaps in nuclei and homogeneous neutron matter [5].

In this paper we generalize the $V_{\text{low } k}$ approach to the hyperon-nucleon (YN) sector. The ultimate goal is to provide effective potentials of similar quality as in the NN case that could serve as the starting point for realistic calculations of the structure of hypernuclei and homogeneous hyperonic matter. At present such a program is hampered by the lack of a comparable data base and the collapse to a unique low-momentum potential is far from obvious. In a first exploratory study we wish to address this point by considering the low-momentum decimation of various potentials by the Nijmegen group. We focus on hyperons with strangeness $S = -1$ for which $I = 1/2$ and $I = 3/2$ isospin states are available. For $I = 1/2$, several hyperon-nucleon channels occur which require new technical developments for the coupled RG flow equations.

The effective, low-momentum potential $V_{\text{low } k}$ for elastic two-body scattering is obtained by integrating out high-momentum components of a realistic bare potential V interaction. This is achieved by imposing a cutoff Λ on all loop integrals in the half-on-shell (HOS) T-matrix equation and replacing the bare potential V with the effective $V_{\text{low } k}$ potential. Since the physical low-energy quantities must not depend on the cutoff, the HOS T-matrix should be preserved for relative three-momenta $k',k \leq \Lambda$. This results in a modified Lippmann-Schwinger equation with a cutoff-dependent effective potential $V_{\text{low } k}$

$$T(k',k;k^2) = V_{\text{low } k}(k',k) + \frac{2}{\pi}\mathcal{P}\int_0^\Lambda q^2 dq \frac{V_{\text{low } k}(k',q)T(q,k;k^2)}{k^2 - q^2}.$$

By demanding $dT(k',k;k^2)/d\Lambda = 0$, an exact renormalization group (RG) flow equation for $V_{\text{low } k}$ can be obtained [6]

$$\frac{d}{d\Lambda}V_{\text{low } k}(k',k) = \frac{2}{\pi}\frac{V_{\text{low } k}(k',\Lambda)T(\Lambda,k;\Lambda^2)}{1 - k^2/\Lambda^2}\ . \qquad (1)$$

Integrating this flow equation with a given initial bare potential at a large cutoff (small distance) one obtains the physically equivalent effective theory ($V_{\text{low } k}$) at a smaller cutoff Λ (larger distance).

Instead of solving the RG equation (1) directly as a differential equation with e.g. standard Runge-Kutta methods, we use the Andreozzi-Lee-Suzuki (ALS) iteration method, which is based on a similarity transformation [7, 8]. With folded diagram techniques it has been shown [9] that this iteration method indeed yields a solution of Eq. (1).

By construction, the resulting $V_{\text{low } k}$ for the NN interaction reproduces the empirical deuteron binding energy and scattering phase shifts up to $E_{lab} = 2\hbar^2\Lambda^2/M$. Most importantly, it is found that for $\Lambda < 2$ fm^{-1} ($E_{lab} < 330$ MeV) $V_{\text{low } k}$ is independent of the particular V_{NN} model, i.e. all diagonal matrix elements of the different high-precision potentials collapse to a single unique low-momentum effective potential [6]. This can be largely attributed to the long-range one-pion exchange (OPE) which is common to all realistic potentials and dominates the low-momentum scattering. The main effect of the RG evolution is a constant shift of the bare matrix elements which removes the ambiguities in the short-range part of the potential.

The energy-independent $V_{\text{low } k}$ is non-Hermitian. This can readily be seen from the RG equation (1) because the momenta are treated asymmetrically. With a second similarity transformation the non-Hermiticity of $V_{\text{low } k}$ can be eliminated. Phase shifts are preserved by this second transformation and there are a number of such phase shift equivalent transformations such as the well-known Okubo one [10]. In the present work we have used the Okubo transformation to obtain the Hermitian $V_{\text{low } k}$. For the NN interaction, the diagonal matrix elements are almost unchanged by the second transformation.

As realistic YN interactions we use the soft-core potentials by the Nijmegen group [11]. They are based on one-boson-exchange models (OBE) of the NN potential and use $SU(3)_F$-symmetry to infer the coupling vertices in the presence of a hyperon. Since the flavor symmetry is broken by the finite quark masses the pertinent coupling strengths have to be adjusted to data. Six different fits are available referred to NSC97a - NSC97f in the following. Each potential comes in two basis representations (isospin- and physical particle-basis). In this paper we work on the isospin basis which was also originally used for the potential construction by the Nijmegen group. Therefore all isomultiplets are degenerate. The corresponding isospin-averaged masses are given by $m_N = 938.9$ MeV, $m_\Lambda = 1115.7$ MeV and $m_\Sigma = 1193.1$ MeV. In the $I = 3/2$ channel no Λ-hyperon is involved.

The different Nijmegen fits describe the known YN cross section data equally well ($\chi^2/N \sim 0.55$) but exhibit differences on a more detailed level. Due to the few available data points (only 35 altogether), the phase shifts and some scattering lengths exhibit large variations for different fits [12]. This is in marked contrast to the NN case where the wealth of the data base allows for high-precision potentials. These differ only in their short-range properties which are not constrained by the available data. The collapse of the low-momentum potential $V_{\text{low } k}$ for the NN interaction to a 'unique' effective potential, observed after the RG decimation [1] is basically driven by the precision of the measured phase shifts. Due to the scarceness and uncertainties in the YN data we cannot expect a unique low-momentum YN $V_{\text{low } k}$ potential.

In addition, for the YN interaction the choice of the cutoff Λ is not so obvious. In this work a cutoff around $\Lambda \sim 500$ MeV is chosen which corresponds to 2.5 fm^{-1} in the configuration space.

We first consider the $I = 3/2$ case which corresponds to the $\Sigma N \to \Sigma N$ channel. The ALS iteration is exactly the same as for the NN interaction albeit with the appropriate substitution of the hyperon mass. For the ALS iteration we have used a cutoff $\Lambda \equiv \Lambda_P = 500$ MeV in the model space (P-space) with 64 grid points and a cutoff $\Lambda_Q = 10$ GeV for the complementary Q-space also with 64 grid points. In order to verify the insensitivity of the

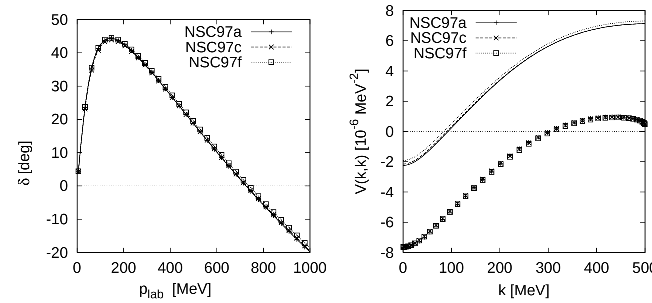

FIG. 1: Left panel: The corresponding $^1S_0$ phase shifts for the six different potentials as a function of the momentum in the lab frame p_{LAB}. Right panel: Three diagonal bare potentials V_{bare} (NSC97a,c,f; dashed lines) and three (Hermitian) $V_{\text{low } k}$ matrix elements (dotted) for $\Sigma N \to \Sigma N$ ($I = 3/2$, $^1S_0$) versus the relative momentum k.

results on these quantities we have varied the grid points in an interval [32, 70] and the cutoff Λ_Q in between the range $\Lambda_Q \pm 2$ GeV. The standard ALS method converges rapidly. Details will be presented in [13].

The results for the $^1S_0$ partial wave are shown in Fig. 1. In the right panel the diagonal matrix elements for three different bare potentials (dashed lines) and the corresponding RG evolved $V_{\text{low } k}$ potentials are displayed versus the relative momentum k. The RG decimation yields a soft-core $V_{\text{low } k}$ potential which is more attractive (dotted curves). As already mentioned, the $V_{\text{low } k}$'s are basically non-Hermitian after the RG decimation. By means of an Okubo-transformation we obtain Hermitian $V_{\text{low } k}$ potentials. As in the NN case [1] the differences are negligible.

By construction, the on-shell T-matrix is phase-shift equivalent which must result in identical phase shifts for a given bare potential fit and momenta below the cutoff Λ. This is demonstrated in the left panel of Fig. 1. This comparison also serves as a test for our numerics. For the $^1S_0$ partial wave, the bare potentials do not differ strongly and thus yield almost the same $V_{\text{low } k}$ for all fits considered. As discussed below, this changes for

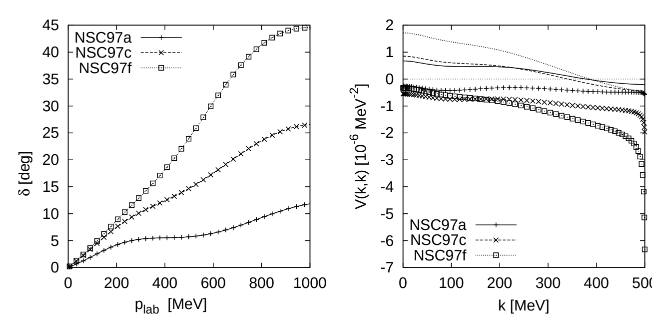

FIG. 2: The same as in Fig. 1 for the partial wave $^3S_1$. Left panel: nuclear bar phase shift.

higher partial waves where the different bare potential fits deviate significantly and therefore no 'unique' $V_{\mathrm{low}\ k}$ is found.

As in the NN case, the YN interaction contains tensor components and hence partial waves can mix. The generalization of the ALS iteration in the presence of tensor forces is straightforward. For $S = 1$ one has to enlarge the T-matrix to a (2×2) block structure in the standard way, corresponding to the orbital angular momentum combinations $L = J \pm 1$. Also for this case we have verified that $V_{\mathrm{low}\ k}$ is phase-shift equivalent to the bare potentials, as is shown in left panel Fig. 2 for the $^3S_1$ partial wave. The right panel displays the corresponding $V_{\mathrm{low}\ k}$ potentials together with the bare ones. They again become more attractive for all Nijmegen fits. Due to strong differences in the phase shifts we do not find a collapse of $V_{\mathrm{low}\ k}$ to one potential, especially at larger momenta where deviations are most pronounced.

The RG flow equation (1) implies a pole at the cutoff boundary $k = \Lambda$. In the vicinity of this pole the slope for the $V_{\mathrm{low}\ k}$ diverges which can be clearly seen in the right panel of Fig. 2.

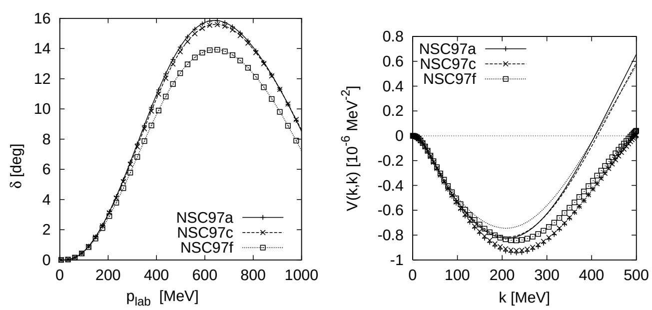

FIG. 3: Same as Fig. 1 for the partial wave $^1P_1$ and $I = 3/2$.

To complete the analysis for the $I = 3/2$ channel we show in Fig. 3 the $^1P_1$ partial wave which serves as an example for the spin singlet-spin triplet transition. Such transitions which are induced by the tensor force are negligible in the NN interaction due to the small mass differences. However, for the YN interaction this is not the case anymore and these transitions can be significant. Due to the smaller deviations in the phase shift, the corresponding $V_{\mathrm{low}\ k}$ interactions collapse to single potential (especially for small momenta).

The treatment of of the $I = 1/2$ channel is much more complicated. For this isospin four coupled channels available corresponding to the transitions: $(\Lambda N, \Sigma N) \to (\Lambda N, \Sigma N)$. This is a completely new situation for the $V_{\mathrm{low}\ k}$ approach. Since now channels with different masses couple, new phenomena are to be expected concerning e.g. the convergence behavior of the ALS iteration.

In flavor space the Lippmann-Schwinger equation becomes a coupled (2×2) matrix equation where the diagonal matrix elements describe respectively the $\Lambda N \to \Lambda N$ and $\Sigma N \to \Sigma N$ channels while the off-diagonal elements describe the $\Lambda N \to \Sigma N$ and $\Sigma N \to \Lambda N$ transitions. Using a notation where we only list the hyperons $(Y, Y' = \Lambda, \Sigma)$

$$T^{Y'Y}(k', k; E_k^Y) = V^{Y'Y}(k', k) + \sum_{Z=\Lambda,\Sigma} \frac{2}{\pi} \mathcal{P} \int_0^\infty dq q^2 \frac{V^{Y'Z}(k', q) T^{ZY}(q, k; E_k^Y)}{E_k^Y - H_0^Z} \quad (2)$$

with the free Hamiltonian $H_0^Y(q) = \frac{q^2}{2\mu_Y} + m_Y + m_N$, the energy $E_k^Y = \frac{k^2}{2\mu_Y} + m_Y + m_N$ and the reduced mass $\mu_Y = \frac{m_Y m_N}{m_Y + m_N}$ we have (with the inclusion of tensor forces) in general four coupled equations to solve. For some channels, the mass difference $m_\Sigma - m_\Lambda$ enters in the denominator of Eq. (2) which induces e.g. a threshold behavior for the Σ hyperon. The mass differences which are not present in the NN interaction enter also in the ALS iteration. It is found that the standard ALS iteration procedure does not converge to the proper consecutive set of eigenvalues. As a consequence, a wrong sorting of the eigenvalues in the different P- and Q-spaces emerges. Using a modified ALS iteration or by introducing an energy cutoff solves this problem and convergence to the correct eigenvalues can be found [13].

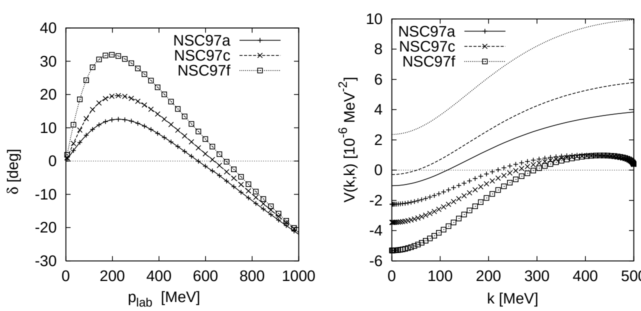

FIG. 4: $^1S_0$ partial wave for the $I = 1/2$ $\Lambda N \to \Lambda N$ channel. Left panel: Corresponding phase shifts. Right panel: bare potentials and $V_{\mathrm{low}\ k}$ potentials. The labeling in both panels is the same as in Fig. 1.

As an example we show in Fig. 4 the $^1S_0$ partial wave for the $\Lambda N \to \Lambda N$ channel. All $V_{\mathrm{low}\ k}$ potentials are again more attractive and a more narrow grouping as compared to the bare potentials can be observed. One also observes that the shift of the $V_{\mathrm{low}\ k}$ potential is largest for the bare NSC97f potential and smallest for the NSC97a potential in contrast to all other partial waves. The $V_{\mathrm{low}\ k}$'s are shifted in such a way that the potentials collapse for relative momenta near the cutoff reflecting the corresponding trends in the phase shifts.

In Fig. 5 the $\Lambda N \to \Lambda N$ channel with tensor coupling is shown for the $^3S_1$ partial wave . Here again the RG

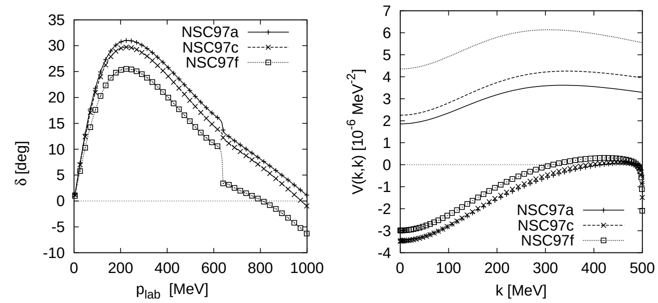

FIG. 5: Same as Fig. 4 but for the partial wave $^3S_1$. Left panel: nuclear bar phase shifts.

decimation pushes the bare potentials down basically by a (large) constant to attractive $V_{\text{low }k}$'s. Since this channel includes the ΣN transition, the Σ threshold is visible in the phase shifts for lab-momenta above 600 MeV. The jump in the phase shift at the threshold depends strongly on the model.

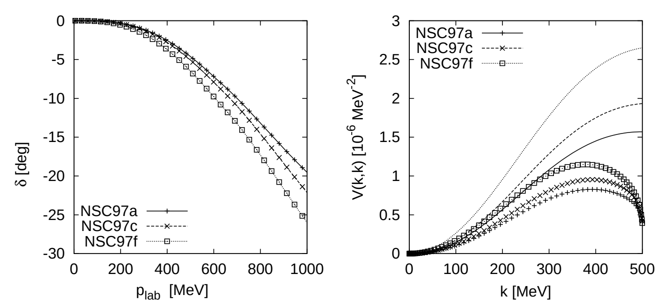

FIG. 6: Same as Fig. 4 but for the partial wave $^1P_1$. Left panel: nuclear bar phase shift.

To complete this analysis results for the spin singlet - spin triplet transition in the $\Lambda N \to \Lambda N$ channel for the $^1P_1$ partial wave are presented in Fig. 6. For small momenta no RG decimation takes place. For this partial wave the bare interaction is repulsive and so is $V_{\text{low }k}$.

Recently, it has been shown that the $V_{\text{low }k}$ approach which is based on RG techniques provides a novel and powerful tool to obtain phase-shift equivalent low-momentum NN interactions. After the RG decimation a unique low-momentum potential $V_{\text{low }k}$ for different high-precision NN interactions was found. This model-independence of the diagonal matrix elements of the $V_{\text{low }k}$ is an important property which is basically driven by the phase shift equivalence of the input models.

In the present work the model-dependence of the $V_{\text{low }k}$ for the YN interaction is investigated. Because of coupled-channel effects in flavor space that are not present in the NN case, the RG evolution is technically more complicated but can be treated. Due to the few experimental data currently available, the model fits by the Nijmegen group do not allow for a unique low-momentum YN interaction. It is therefore of importance to calculate $V_{\text{low }k}$ for other YN models such as the revised Jülich potential [14]. Such calculations are in preparation [13].

Although no unique YN low-momentum interaction is obtained at present, a convergence of the different $V_{\text{low }k}$'s is seen generally for all the Nijmegen potentials. Especially, for partial waves which do not deviate strongly for different bare potentials the uniqueness of $V_{\text{low }k}$ is pronounced. All $V_{\text{low }k}$ potentials are much softer than the bare ones. Softer interactions lead to stronger binding which should be of relevance in microscopic hypernuclei calculations.

By construction, all low-energy two-body observables are cutoff-independent. Bogner et al. argue that any (new) induced cutoff dependence is due to higher-body forces [2]. For the NN interaction they conclude that such contributions are rather small. For the YN interaction this is still an open issue and should be tested in light hypernuclei.

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* E-Mail:bernd-jochen.schaefer@physik.tu-darmstadt.de

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